$N = 8$ superfield formulation of the Bagger-Lambert-Gustavsson model

Martin Cederwall

Fundamental Physics, Chalmers University of Technology
SE 412 96 Göteborg, Sweden
E-mail: `martin.cederwall@chalmers.se`

ABSTRACT: We reformulate the Bagger-Lambert-Gustavsson model using an $N = 8$ superspace, thus making the full supersymmetry manifest. The formulation is based on appropriate "pure spinor wave functions" for the Chern-Simons and matter multiplets. The Lagrangian has an extremely simple structure, essentially containing a Chern-Simons-like term for the gauge field wave function and a minimally coupled matter wave function. No higher order interactions than cubic are present. The consistency of the setup relies on an interplay between the algebraic structures of the 3-algebra and of the pure spinors.

KEYWORDS: Extended Supersymmetry, Conformal Field Models in String Theory, BRST Symmetry.

The Bagger-Lambert-Gustavsson (BLG) model [1–4] is a maximally ($N = 8$) supersymmetric and conformal interacting 3-dimensional model, whose local degrees of freedom consist of scalar multiplets. It has been proposed to be related to AdS$_4$ boundary theories describing multiple membrane configurations, although the interpretation is unclear, partly due to the fact that there only is one unique finite-dimensional representation for the fields (an $so(4)$ gauge algebra with matter in the vector representation) [5, 6] that limits its use to two-membranes stacks [7, 8]. The discovery of the BLG model has been followed by an intense activity concerning its interpretation and possible modifications (either with degenerate or indefinite metric for the scalars or with less supersymmetry), possibly relevant for the formulation of multiple membrane theory The literature in these directions of the subject is large, and we refer to e.g. ref. [9] for a selection of references.

It is of course desirable to formulate a model with as much manifest symmetry as possible. Being a maximally supersymmetric theory, the BLG model has on-shell supersymmetry, and no finite set of auxiliary fields. An appropriate treatment of such models — the standard example being $D = 10$ super-Yang-Mills — is to use pure spinors. This is the approach that will be taken in this letter. We will in fact investigate the most general form of the interactions, under very mild assumptions. As expected, we recover the condition consisting of the existence of a 3-algebra and its structure, although this derivation becomes much easier when supersymmetry is kept manifest. It was realised early that pure spinor techniques are relevant for supersymmetric theories and supergravity [10–12]. The principles behind and applications of pure spinors for maximally supersymmetric field theories may be found in e.g. refs. [13–17].

The first thing to decide is what the pure spinors are, and to find the correct representations of the wave functions. The spinorial coordinates are $\theta^{A\alpha}$, where $A = 1, 2$ is a spinor index under the 3-dimensional Lorentz group Spin$(1, 2) \approx$ SL$(2, \mathbb{R})$ and $\alpha = 1, \ldots, 8$ is a chiral spinor $\mathbf{8}_s$ under the R-symmetry group Spin(8). We denote the 3-dimensional vector indices $a, b, \ldots$. The superspace torsion is $T^c_{A\alpha, B\beta} = \gamma^c_{AB} \delta_{\alpha\beta}$. A pure spinor BRST operator is generically formed as $Q = \lambda^{A\alpha} D_{A\alpha}$, and the purpose of the pure spinor constraint is to make Q nilpotent by projecting out the torsion in Q^2. We see that the appropriate pure spinor constraint is

$$(\lambda^{(A} \lambda^{B)}) = 0 \,. \tag{1}$$

The notation for spinor contractions, denoted by parentheses, is throughout the paper that Spin(8) indices are contracted, while SL(2) indices are kept explicit. Very similar pure spinor constraints have been considered in ref. [18]. If we introduce Dynkin labels for representations of $sl(2) \oplus so(8)$, θ, D and λ transform in $\mathbf{8}_s = (1)(0010)$. A bilinear in λ contains the representations $(0)(0100) \oplus (2)(0000) \oplus (2)(0020)$, and the second of these (the 3-dimensional vector) is removed by the pure spinor constraint. The pure spinor is a (first quantised) ghost, with ghost number 1. A pure spinor wave function is seen as a power expansion in λ. In order to calculate cohomologies of Q we need a list of the representations occurring at any power λ^n. Using the pure spinor constraint (1), one finds

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$
dim $=0$	(0)(0000)				
$\frac{1}{2}$	•	•			
1	•	(2)(0000)	•		
$\frac{3}{2}$	•	•	•	•	
2	•	•	(2)(0000)	•	•
$\frac{5}{2}$	•	•	•	•	•
3	•	•	•	(0)(0000)	•
$\frac{7}{2}$	•	•	•	•	•
4	•	•	•	•	•

Table 1: The cohomology of the scalar complex.

that these representations are

$$\bigoplus_{i=0}^{[n/2]} (n-2i)(0,i,n-2i,0) \,. \tag{2}$$

This looks at first sight more complicated than the situation in e.g. $D=10$, $N=1$ pure spinor space [19], where one has one irreducible representation at each n. Irreducible representations however occur in many other situations, like $D=11$ supergravity [15, 20–22], and we will see that the end results, the cohomologies, are quite simple.

Let us first consider a scalar wave function. Its expansion in λ simply contains the representations in eq. (2) of decreasing ghost number $1-n$. In order to find the representations of component fields (and field equations) one considers, as usual, the cohomology of Q at zero modes of ∂_a. This is a purely algebraic computation, that can be performed by hand, but preferably by the computer-based method of ref. [15]. The result is given in table 1.

The grading n is the degree of homogeneity in λ, and the vertical direction is the expansion in the fermionic coordinates. The superfields at different n are shifted vertically in the table so that Q acts horizontally. This cohomology describes a Chern-Simons field, its ghost and the associated anti-fields. The fermionic scalar wave function Ψ has to be assigned ghost number 1 and dimension 0 in order for the connection A_a to have ghost number 0 and dimension 1. It is essential to note that the antifield A_a^* has dimension 2, so that the equation of motion for A following from $Q\Psi=0$ is first order in derivatives. It reads $dA=0$. There is no additional input to the pure spinor formalism that gives the Chern-Simons dynamics (as opposed to e.g. Yang-Mills). The full non-linear Chern-Simons theory follows from the non-linear modification of the cohomology:

$$Q\Psi + \frac{1}{2}[\Psi,\Psi] = 0 \,, \tag{3}$$

where $[\cdot,\cdot]$ is the Lie bracket.

We also need to describe the matter multiplet. It contains scalars ϕ^I in the vector representation $(\mathbf{1},\mathbf{8}_v) = (0)(1000)$ of Spin(8) and spinors $\chi^{A\dot\alpha}$ in the chiral representation

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$
$\dim = \frac{1}{2}$	$(0)(1000)$				
1	$(1)(0001)$	$\bullet$			
$\frac{3}{2}$	$\bullet$	$\bullet$	$\bullet$		
2	$\bullet$	$(1)(0001)$	$\bullet$	$\bullet$	
$\frac{5}{2}$	$\bullet$	$(0)(1000)$	$\bullet$	$\bullet$	$\bullet$
3	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
$\frac{7}{2}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$

Table 2: The cohomology of the vector complex.

$(\mathbf{2}, \mathbf{8}_c) = (1)(0001)$. Since there is no gauge symmetry, we expect the scalar fields to sit at the lowest order of λ in the wave function. We therefore try a bosonic wave function Φ^I of dimension $1/2$ and ghost number 0 in $(0)(1000)$. A pure spinor wave function in a non-scalar representation of the structure group (such as supergravity complexes containing the spinorial 1-form frame field, or some non-maximally supersymmetric models [23]) is always subject to some further condition; the cohomology would otherwise just be the tensor product of the wave function representation with the cohomology of a scalar wave function. These extra conditions typically remove "smaller representations", in the same spirit as the pure spinor constraint itself. In the present case one may postulate an additional invariance under

$$\delta_\varrho \Phi^I = (\lambda^A \sigma^I \varrho_A) \tag{4}$$

for arbitrary functions $\varrho_A^{\dot\alpha}$. The effect of this further invariance (which should be implemented the same way as the pure spinor constraint, in the sense that the wave function belongs to an equivalence class modulo such functions) is that the expansion of Φ^I at order λ^n (i.e., ghost number $-n$) contains the representations

$$\bigoplus_{i=0}^{[n/2]} (n-2i)(1, i, n-2i, 0). \tag{5}$$

Note the similarity to the representation content of the scalar case (2), the difference is only the 1 in the vector position. This construction is vindicated by the observation that the zero-mode cohomology is the correct one, given in table 2. We observe that the antifields have the correct dimensions and representations. The equations of motion derived from $Q\Phi^I = 0$ of course are $\Box\phi^I = 0$, $(\gamma^a \partial_a \chi)^{A\dot\alpha} = 0$.

Before introducing interactions, we would like to discuss how to write an action. Consider first the Chern-Simons multiplet. The cohomology contains a singlet at $\lambda^3 \theta^3$, the position of the antighost. This cohomology can serve as a measure with ghost number -3. Strictly speaking, this is not true with the minimal set of variables described here (analogous to the single pure spinor in $D = 10$). Unless further variables are introduced, this measure is degenerate, since an action constructed from it will not contain components of fields at higher powers than λ^3. Berkovits has shown how to add extra variables that render the measure non-degenerate without changing the cohomology [24]. We will not go

further into this procedure, but note that it is clear that the corresponding construction works also in the present setting. Using a non-degenerate measure in non-minimal pure spinor space, one may write a Lagrangian for the Chern-Simons multiplet:

$$\mathcal{L}_{\text{CS}} = \frac{1}{2} < \Psi, Q\Psi + \frac{1}{3}[\Psi, \Psi] > \,, \tag{6}$$

where $< \cdot, \cdot >$ is a trace on the Lie algebra and $[\cdot, \cdot]$ the Lie bracket.

How does one write a (linearised) Lagrangian for the scalar multiplet? There are two issues, that turn out to be solved simultaneously. The wave function Φ^I is bosonic, which excludes an expression like $\Phi^I Q\Phi^I$, rather one needs to contract the SO(8) vector indices with some antisymmetric tensor. In addition, to get ghost number 3 (there is no other scalar cohomology available), one needs an insertion of two powers of λ. The unique possibility seems to be

$$\mathcal{L}_{\text{free scalar}} = \frac{1}{2} M_{IJ} \Phi^I Q\Phi^J \,, \tag{7}$$

with $M_{IJ} = \varepsilon_{AB}(\lambda^A \sigma_{IJ} \lambda^B)$. One now has to check that the equations of motion $M_{IJ} Q\Phi^J = 0$ are equivalent to $Q\Phi^I = 0$. This happens to be true exactly thanks to the invariance (4). Namely, any part of Φ^I of the form $(\lambda^A \sigma^I \varrho_A)$ drops out of $M_{IJ}\Phi^J$ due to the Fierz identity $\varepsilon_{AB}(\lambda^A \sigma_{IJ} \lambda^B)(\sigma^J \lambda^C)^{\dot{a}} = 0$, which easily is shown to hold for pure spinors (but not general ones). This shows that the form (7) of the scalar field Lagrangian is good, and gives yet another reason for the choice of the content of the wave function implied by the equivalence classes of eq. (4). The factor of λ^2 in eq. (7), whose necessity we have already given a number of arguments for, will turn out to be crucial in checking the consistency of the interacting Lagrangian. This new mechanism, with insertions of λ's in the action and the corresponding consistent modding out of representations in the wave function, may turn out to have applications in different settings, e.g. in the context of supergravity.

We now have a working supersymmetric description of the non-interacting fields (in the Chern-Simons case self-interaction is included). The next step is to let the scalars transform also under some representation $\mathbf{R}$ of the Lie algebra of Ψ. We introduce traces $< \cdot, \cdot >_{\mathbf{adj}}$ for the adjoint and $< \cdot, \cdot >_{\mathbf{R}}$ for $\mathbf{R}$, which has to be an orthogonal representation. The Lie bracket is written $[\cdot, \cdot]$ and the action of an element $T \in \mathbf{adj}$ on $x \in \mathbf{R}$ is denoted $T \cdot x$. We have the obvious relations $< x, T \cdot y >_{\mathbf{R}} = - < T \cdot x, y >_{\mathbf{R}}$ and $T \cdot (U \cdot x) - U \cdot (T \cdot x) = [T, U] \cdot x$.

One term in the Lagrangian, apart from the ones already discussed, comes from the "covariantisation" of $Q\Phi^I$ to $(Q + \Psi \cdot)\Phi^I$. We then have

$$\mathcal{L} = < \Psi, Q\Psi + \frac{1}{3}[\Psi, \Psi] >_{\mathbf{adj}} + \frac{1}{2} M_{IJ} < \Phi^I, Q\Phi^J + \Psi \cdot \Phi^J >_{\mathbf{R}} \,. \tag{8}$$

When trying to find other terms for an Ansatz with dimension 0 and ghost number 3, one finds that the terms already present in eq. (8) exhaust the list of possible ones, as long as no dimensionful constant is introduced and no explicit fermionic derivatives or derivatives with respect to λ are allowed to enter (neither of these are wanted, unless we search for higher derivative modifications). Essentially, counting of dimension demands one power of λ for each Φ. Unless the number of Φ's is even, one can not form a scalar. Ghost number counting then limits the number of λ's, and consequently of Φ's, to zero or two. It turns

out that the Lagrangian (8) is the full answer, but before making that claim we have to examine its consistency.

All fields are obtained from cohomologies, i.e., free fields are solutions of "$Q(\text{field}) = 0$" modulo gauge transformations "$\delta(\text{field}) = Q(\text{gauge parameter})$". When interactions are turned on, there still has to exist a gauge invariance of the form "$\delta(\text{field}) = Q(\text{gauge parameter}) + (\text{interaction terms})$". This applies also to the wave function for the scalars. Consistency of the interactions is proven if one finds the invariance under such gauge transformations. This should be equivalent to demanding that the action satisfies a Batalin-Vilkovisky master equation (it would be interesting to find explicit generic expressions for the anti-bracket in the pure spinor framework).

The gauge invariance corresponding to the Chern-Simons field is the simplest part. The extra "connection term" in the minimal coupling of the matter multiplet was introduced to ensure this invariance, and the action is indeed (almost manifestly) invariant under

$$\delta_\Lambda \Psi = Q\Psi - [\Lambda, \Psi] \, , \tag{9}$$

$$\delta_\Lambda \Phi^I = -\Lambda \cdot \Phi^I \, , \tag{10}$$

up to a Q-exact term (a "total derivative").

A general variation of the Lagrangian is

$$\delta\mathcal{L} = <\delta\Psi, Q\Psi + \frac{1}{2}[\Psi, \Psi] + \frac{1}{2}M_{IJ}\{\Phi^I, \Phi^J\} >_{\mathbf{adj}} + M_{IJ} <\delta\Phi^I, Q\Phi^J + \Psi \cdot \Phi^J >_{\mathbf{R}} \, , \tag{11}$$

where we have introduced the notation $\{\cdot, \cdot\}$ for the formation of an adjoint element from the antisymmetric product of two elements in $\mathbf{R}$ via $<x, T \cdot y>_{\mathbf{R}} = <T, \{x, y\}>_{\mathbf{adj}}$. The gauge transformation corresponding to the matter wave function is

$$\delta_\Xi \Psi = -M_{IJ}\{\Phi^I, \Xi^J\} \, , \tag{12}$$

$$\delta_\Xi \Phi^I = Q\Xi^I + \Psi \cdot \Xi^I \, . \tag{13}$$

Roughly speaking, the second of these equations is like a covariant derivative. When applied to the covariantised matter kinetic term a field strength $(Q + \Psi)^2$ arises, and this is cancelled by the appropriate transformation of Ψ, whose Chern-Simons term varies to the field strength. The only term not immediately cancelled comes from the variation of Ψ in the matter kinetic term. It contains four powers of λ and is proportional to

$$M_{IJ}M_{KL} <\{\Phi^I, \Phi^J\}, \{\Phi^K, \Xi^L\}>_{\mathbf{adj}} \, . \tag{14}$$

For general choices of the representation $\mathbf{R}$ and of $<\cdot, \cdot>_{\mathbf{adj}}$ this term will not vanish, and there is no consistent interaction. The pure spinors give information on allowed structures. The fourth power of a pure spinor contains the representations $(0)(0200) \oplus (2)(0120) \oplus (4)(0040)$. This means that the product $M_{IJ}M_{KL}$, being $\mathrm{Spin}(1, 2)$ scalar, is in $(0)(0200)$, and as a consequence $M_{[IJ}M_{KL]} = 0$. If, and only if, the expression $<\{x, y\}, \{z, w\}>_{\mathbf{adj}}$ is completely antisymmetric in its arguments, the potentially problematic term given by eq. (14) vanishes. It is then convenient to introduce the antisymmetric 3-bracket $[\![\cdot, \cdot, \cdot]\!]$ via $[\![a, b, c]\!] = \{a, b\} \cdot c$, or equivalently, $<\{x, a\}, \{b, c\}>_{\mathbf{adj}} = <x, [\![a, b, c]\!]>_{\mathbf{R}}$.

It is interesting to examine the (super-)algebra of gauge transformations. The commutators $[\delta_\Lambda, \delta_{\Lambda'}]$ and $[\delta_\Lambda, \delta_\Xi]$ are "covariant". The remaining one, $[\delta_\Xi, \delta_{\Xi'}]$, requires calculation, which when acting on Ψ yields $[\delta_\Xi, \delta_{\Xi'}]\Psi = \delta_{\Lambda(\Xi, \Xi')}\Psi$, where $\Lambda(\Xi, \Xi') = -M_{IJ}\{\Xi, \Xi'\}$. The structure is like a superalgebra, where the anticommutator of two fermionic gauge transformations gives a bosonic one. Performing the same calculation on Φ^I gives $[\delta_\Xi, \delta_{\Xi'}]\Phi^I = -2M_{JK}[\![\Xi^{[K}, \Xi'^{I]}, \Phi^J]\!] = \delta_{\Lambda(\Xi, \Xi')}\Phi^I - 3M_{JK}[\![\Xi^{[I}, \Xi'^J, \Phi^{K]}]\!]$. The last term is an equivalence transformation of the type (4). Any transformation $\delta\Phi^I = s^{IJK}M_{JK}$ with antisymmetric s^{IJK} is trivial, as seen by setting the parameter $\varrho \sim s^{IJK}\sigma_{IJK}\lambda$.

We have thus established that the existence of the 3-algebra is the possibility allowed by the pure spinors for the modified (interacting) cohomology to exist. The 3-bracket of course has to satisfy the fundamental identity, but it provides no further information, as it follows from the structure already defined (more precisely, from the fact that the the 3-bracket of three elements in **R** itself transforms in **R** under a transformation with an element in **adj** defined by two elements in **R** via the curly bracket). Once the structure of the pure spinor wave functions is established, the calculation is considerably simpler than in the component formalism — apart from a single term giving the information about the need of an antisymmetric 3-bracket the symmetries are essentially covariantly realised. A striking property of the Lagrangian is that it only contains terms quadratic and cubic in fields, and that it, in strong contrast to the component action, essentially consists of a Chern-Simons term and minimally coupled matter.

The component action contains a 6-point coupling of scalars and a coupling of two scalars and two fermions, appropriate for a conformal model. The interactions between component fields arise after elimination of auxiliary fields, much in the same way as the $D = 10$ super-Yang-Mills dynamics, with 4-point couplings, arises from a Chern-Simons-like action for the pure spinor wave function. This calculation is of standard superspace type and goes schematically as follows. Let ϕ^I be the ghost number 0 part of Φ^I. The matter equations of motion then all follow from the equation $\mathcal{D}\phi|_{(1)(1010)} = 0$, where $\mathcal{D}$ includes the ghost number 0 gauge superfield. This is solved by $\mathcal{D}\phi = \mathcal{D}\phi|_{(1)(0001)} = \chi$. Acting with another fermionic covariant derivative yields $\mathcal{D}\chi = \mathcal{D}^2\phi = (\partial_A + f)\phi = \partial_A\phi + \phi^3$, where f is the field strength with two fermionic components, which by the equation of motion for the gauge superfield is proportional to ϕ^2. Yet another fermionic derivative gives the equation of motion for χ, schematically reading $\bar{\partial}_A\chi + \phi^2\chi = 0$, etc. The formalism also makes it completely clear why the Chern-Simons gauge field is needed to close the supersymmetry algebra in the component formulation: the interactions between matter fields arise from elimination of the dimension-1 superspace component $f_{A\alpha, B\beta}$ of the field strength.

Acknowledgments

References

[1] J. Bagger and N. Lambert, *Modeling multiple M2's*, *Phys. Rev.* **D 75** (2007) 045020 [hep-th/0611108].

[2] A. Gustavsson, *Algebraic structures on parallel M2-branes*, `arXiv:0709.1260`.

[3] J. Bagger and N. Lambert, *Gauge symmetry and supersymmetry of multiple M2-branes*, *Phys. Rev.* **D 77** (2008) 065008 [`arXiv:0711.0955`].

[4] J. Bagger and N. Lambert, *Comments on multiple M2-branes*, *JHEP* **02** (2008) 105 [`arXiv:0712.3738`].

[5] G. Papadopoulos, *M2-branes, 3-Lie algebras and Plücker relations*, *JHEP* **05** (2008) 054 [`arXiv:0804.2662`].

[6] J.P. Gauntlett and J.B. Gutowski, *Constraining maximally supersymmetric membrane actions*, `arXiv:0804.3078`.

[7] N. Lambert and D. Tong, *Membranes on an orbifold*, `arXiv:0804.1114`.

[8] J. Distler, S. Mukhi, C. Papageorgakis and M. van Raamsdonk, *M2-branes on M-folds*, *JHEP* **05** (2008) 038 [`arXiv:0804.1256`].

[9] N. Lambert, *Lagrangians for multiple M2-branes*, talk at *STRINGS* 2008, August 2008, CERN, Ginevra, Switzerland (2008);
J. Maldacena, *N = 6 Chern Simons matter theories, M2 branes and supergravity*, talk at *STRINGS* 2008, August 2008, CERN, Ginevra, Switzerland (2008);
S. Mukhi, *Multiple membrane dynamics*, talk at *STRINGS* 2008, August 2008, CERN, Ginevra, Switzerland (2008);
see www.cern.ch/strings2008.

[10] B.E.W. Nilsson, *Pure spinors as auxiliary fields in the ten-dimensional supersymmetric Yang-Mills theory*, *Class. and Quant. Grav.* **3** (1986) L41.

[11] P.S. Howe, *Pure spinor lines in superspace and ten-dimensional supersymmetric theories*, *Phys. Lett.* **B 258** (1991) 141 [*Addendum ibid.* **259** (1991) 511].

[12] P.S. Howe, *Pure spinors, function superspaces and supergravity theories in ten and eleven dimensions*, *Phys. Lett.* **B 273** (1991) 90.

[13] M. Cederwall, B.E.W. Nilsson and D. Tsimpis, *The structure of maximally supersymmetric super-Yang-Mills theory-constraining higher order corrections*, *JHEP* **06** (2001) 034 [`hep-th/0102009`].

[14] N. Berkovits, *Covariant quantization of the superparticle using pure spinors*, *JHEP* **09** (2001) 016 [`hep-th/0105050`].

[15] M. Cederwall, B.E.W. Nilsson and D. Tsimpis, *Spinorial cohomology and maximally supersymmetric theories*, *JHEP* **02** (2002) 009 [`hep-th/0110069`];
M. Cederwall, *Superspace methods in string theory, supergravity and gauge theory*, in *Lectures at the XXXVII Winter School in Theoretical Physics "New Developments in Fundamental Interactions Theories"*, February 6–15, Karpacz, Poland (2001), `hep-th/0105176`.

[16] M. Cederwall, B.E.W. Nilsson and D. Tsimpis, *D = 10 super-Yang-Mills at $O(\alpha'^2)$*, *JHEP* **07** (2001) 042 [`hep-th/0104236`].

[17] M. Movshev and A. Schwarz, *On maximally supersymmetric Yang-Mills theories*, *Nucl. Phys.* **B 681** (2004) 324 [`hep-th/0311132`].

[18] P. Fré and P.A. Grassi, *Pure spinor formalism for* OSp($N|4$) *backgrounds*, `arXiv:0807.0044`.

[19] N. Berkovits, *Super-Poincaré covariant quantization of the superstring*, *JHEP* **04** (2000) 018 [hep-th/0001035].

[20] M. Cederwall, U. Gran, M. Nielsen and B.E.W. Nilsson, *Manifestly supersymmetric M-theory*, *JHEP* **10** (2000) 041 [hep-th/0007035]; *Generalised 11-dimensional supergravity*, hep-th/0010042.

[21] M. Cederwall, U. Gran, B.E.W. Nilsson and D. Tsimpis, *Supersymmetric corrections to eleven-dimensional supergravity*, *JHEP* **05** (2005) 052 [hep-th/0409107].

[22] P.S. Howe and D. Tsimpis, *On higher order corrections in M theory*, *JHEP* **09** (2003) 038 [hep-th/0305129].

[23] M. Cederwall and B.E.W. Nilsson, *Pure spinors and $D = 6$ super-Yang-Mills*, arXiv:0801.1428.

[24] N. Berkovits, *Pure spinor formalism as an $N = 2$ topological string*, *JHEP* **10** (2005) 089 [hep-th/0509120].